Recitation 10

1. Let X have the exponential distribution with parameter β. Suppose that we wish to test the hypotheses $H_0 : \beta \geq 1$ versus $H_1 : \beta < 1$. Consider the test procedure δ that rejects H_0 if $X \geq 1$.
   (a) Determine the power function of the test.
   (b) Compute the size of the test.

2. Suppose that $X_1, \ldots, X_n$ form a random sample from the uniform distribution on the interval $[0, \theta]$, and that the following hypotheses are to be tested:

$$\begin{aligned} H_0 : &\quad \theta \geq 2, \\ H_1 : &\quad \theta < 2. \end{aligned}$$

   Let $Y_n = \max \{X_1, \ldots, X_n\}$, and consider a test procedure such that the critical region contains all the outcomes for which $Y_n \leq 1.5$.
   (a) Determine the power function of the test.
   (b) Compute the size of the test.

3. Suppose that the proportion p of defective items in a large population of items is unknown, and that it is desired to test the following hypotheses:

$$\begin{aligned} H_0 : &\quad p = 0.2, \\ H_1 : &\quad p \neq 0.2. \end{aligned}$$

   Suppose also that a random sample of 20 items is drawn from the population. Let Y denote the number of defective items in the sample, and consider a test procedure δ such that the critical region contains all the outcomes for which either $Y \geq 7$ or $Y \leq 1$.
   (a) Determine the value of the power function $\pi(p \mid \delta)$ at the points $p = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$, and 1 ; sketch the power function.
   (b) Determine the size of the test.

4. Suppose that $X_1, \ldots, X_n$ form a random sample from the normal distribution with unknown mean μ and known variance 1 . Suppose also that μ_0 is a certain specified number, and that the following hypotheses are to be tested:

$$\begin{aligned} H_0 : &\quad \mu = \mu_0, \\ H_1 : &\quad \mu \neq \mu_0. \end{aligned}$$

   Finally, suppose that the sample size n is 25 , and consider a test procedure such that H_0 is to be rejected if $\left|\bar{X}_n - \mu_0\right| \geq$ c. Determine the value of c such that the size of the test will be 0.05 .

5. Suppose that a certain drug A was administered to eight patients selected at random, and after a fixed time period. the concentration of the drug in certain body cells of each patient was measured in appropriate units Suppose that these concentrations for the eight patients were found to be as follows:

$$1.23, 1.42, 1.41, 1.62, 1.55, 1.51, 1.60, \text{ and } 1.76.$$

   Suppose also that a second drug B was administered to six different patients selected at random, and when the concentration of drug B was measured in a similar way for these six patients, the results were as follows:

$$1.76, 1.41, 1.87, 1.49, 1.67, \text{ and } 1.81.$$

   Assuming that all the observations have a normal distribution with a common unknown variance, test the following hypotheses at the level of significance 0.10 : The null hypothesis is that the mean concentration of drug A among all patients is at least as large as the mean concentration of drug B. The alternative hypothesis is that the mean concentration of drug B is larger than that of drug A.

6. Bottles of a popular cola drink are supposed to contain 300 milliliters (ml) of cola. There is some variation from bottle to bottle because the filling machinery is not perfectly precise. The distribution of contents is normal, with known variance $\sigma^2 = 9$. A student measures the contents of 6 bottles. The results are $299.4, 297.7, 301.0, 298.9, 300.2, 297.0$, with sample mean equal to 299.03. Is this convincing evidence that the mean contents μ of cola bottles is different than the advertised 300ml, at level $\alpha = 0.05$?

7. Suppose that a random variable X has the F distribution with three and eight degrees of freedom. Determine the value of c such that $\Pr(X > c) = 0.975$.

8. Consider two different normal distributions for which both the means μ_1 and μ_2 and the variances σ_1^2 and σ_2^2 are unknown, and suppose that it is desired to test the following hypotheses:

$$\begin{aligned} H_0 : &\quad \sigma_1^2 \leq \sigma_2^2, \\ H_1 : &\quad \sigma_1^2 > \sigma_2^2. \end{aligned}$$

   Suppose further that a random sample consisting of 16ob servations for the first normal distribution yields the values $\sum_{i=1}^{16} X_i = 84$ and $\sum_{i=1}^{16} X_i^2 = 563$, and an independent random sample consisting of 10 observations from the second normal distribution yields the values $\sum_{i=1}^{10} Y_i = 18$ and $\sum_{i=1}^{10} Y_i^2 = 72$.

   (a) What are the M.L.E.'s of σ_1^2 and σ_2^2 ?

   (b) If an F test is carried out at the level of significance 0.05 , is the hypothesis H_0 rejected or not?