Math Recap

1. Express $9x^2 + 12x + 10$ in the form $a(x+b)^2 + c$, where a, b and c are not functions of x.
2. Express the following expressions in terms of $\log(x)$ and $\log(y)$:
   (a) $\log\left(x^3\right)$
   (b) $\log\left(1/x^2\right)$
   (c) $\log(x/y)$
   (d) $\log(xy)$
3. Evaluate
   (a) $\sum_{k=0}^{10} \frac{1}{4^k}$
   (b) $\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{24} + \frac{1}{48} + \cdots$
4. Use integration by parts to evaluate $\int_0^1 x \exp(x) dx$.
5. Differentiate: (a) $\exp\left(-x^2\right)$ (b) $\log(x)$ (c) $\log\left(x^3\right)$ (d) $x^3 \exp(-x)$ (e) $\frac{x}{\exp(x)}$
6. Use integration by substitution so show that $\int_0^1 \frac{1}{\sqrt{z}(1+\sqrt{z})} dz = 2\log(2)$
7. Sketch rough plots of the following functions, indicating at least the point of intersection with the y axis (a) $f(x) = \exp(-x), -\infty < x < \infty$ (b) $f(x) = x^2, -\infty < x < \infty$ (c) $f(x) = \exp\left(-x^2\right), -\infty < x < \infty$

Probability Recap

1. Suppose that $A \subset B$. Show that $B^c \subset A^c$.
2. Suppose that one card is to be selected from a deck of 20 cards that contains 10 red cards numbered from 1 to 10 and 10 blue cards numbered from 1 to 10 . Let A be the event that a card with an even number is selected, let B be the event that a blue card is selected, and let C be the event that a card with a number less than 5 is selected. Describe the sample space S and describe each of the following events both in words and as subsets of S : (a) $A \cap B \cap C$ (b) $B \cap C^c$ (c) $A \cup B \cup C$ (d) $A \cap (B \cup C)$ (e) $A^c \cap B^c \cap C^c$.
3. One ball is to be selected from a box containing red, white, blue, yellow, and green balls. If the probability that the selected ball will be red is $1/5$ and the probability that it will be white is $2/5$, what is the probability that it will be blue, yellow, or green?
4. If the probability that student A will fail a certain statistics examination is 0.5 , the probability that student B will fail the examination is 0.2 , and the probability that both student A and student B will fail the examination is 0.1, what is the probability that at least one of these two students will fail the examination?
5. A point (x, y) is to be selected from the square S containing all points (x, y) such that $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Suppose that the probability that the selected point will belong to each specified subset of S is equal to the area of that subset. Find the probability of each of the following subsets: (a) the subset of points such that $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 \geq \frac{1}{4}$; (b) the subset of points such that $\frac{1}{2} < x + y < \frac{3}{2}$; (c) the subset of points such that $y \leq 1 - x^2$; (d) the subset of points such that $x = y$.
6. If two balanced dice are rolled, what is the probability that the sum of the two numbers that appear will be odd?
7. If a man has six different sportshirts and four different pairs of slacks, how many different combinations can he wear?