8.4 The t Distributions

2. Suppose that $X_1, \dots, X_n$ form a random sample from the normal distribution with unknown mean μ and unknown standard deviation σ, and let $\hat{\mu}$ and $\hat{\sigma}$ denote the M.L.E.'s of μ and σ. For the sample size $n = 17$, find a value of k such that

$$\Pr(\hat{\mu} > \mu + k\hat{\sigma}) = 0.95.$$

8.5 Confidence Intervals

2. Suppose that a random sample of eight observations is taken from the normal distribution with unknown mean μ and unknown variance σ^2, and that the observed values are 3.1, 3.5, 2.6, 3.4, 3.8, 3.0, 2.9, and 2.2. Find the shortest confidence interval for μ with each of the following three confidence coefficients: **(a)** 0.90, **(b)** 0.95, and **(c)** 0.99.

3. Suppose that $X_1, \dots, X_n$ form a random sample from the normal distribution with unknown mean μ and unknown variance σ^2, and let the random variable L denote the length of the shortest confidence interval for μ that can be constructed from the observed values in the sample. Find the value of $E(L^2)$ for the following values of the sample size n and the confidence coefficient γ:

a. $n = 5, \gamma = 0.95$ **d.** $n = 8, \gamma = 0.90$

b. $n = 10, \gamma = 0.95$ **e.** $n = 8, \gamma = 0.95$

c. $n = 30, \gamma = 0.95$ **f.** $n = 8, \gamma = 0.99$

4. Suppose that $X_1, \dots, X_n$ form a random sample from the normal distribution with unknown mean μ and known variance σ^2. How large a random sample must be taken in order that there will be a confidence interval for μ with confidence coefficient 0.95 and length less than 0.01σ?

7. In the June 1986 issue of *Consumer Reports*, some data on the calorie content of beef hot dogs is given. Here are the numbers of calories in 20 different hot dog brands:

186, 181, 176, 149, 184, 190, 158, 139, 175, 148,
152, 111, 141, 153, 190, 157, 131, 149, 135, 132.

Assume that these numbers are the observed values from a random sample of twenty independent normal random variables with mean μ and variance σ^2, both unknown. Find a 90% confidence interval for the mean number of calories μ.

9.1 Problems of Testing Hypotheses

3. Suppose that the proportion p of defective items in a large population of items is unknown, and that it is desired to test the following hypotheses:

$$H_0: \quad p = 0.2,$$
$$H_1: \quad p \neq 0.2.$$

Suppose also that a random sample of 20 items is drawn from the population. Let Y denote the number of defective items in the sample, and consider a test procedure δ such that the critical region contains all the outcomes for which either $Y \geq 7$ or $Y \leq 1$.

a. Determine the value of the power function $\pi(p|\delta)$ at the points $p = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9,$ and 1; sketch the power function.

b. Determine the size of the test.

4. Suppose that $X_1, \dots, X_n$ form a random sample from the normal distribution with unknown mean μ and known variance 1. Suppose also that μ_0 is a certain specified number, and that the following hypotheses are to be tested:

$$H_0: \quad \mu = \mu_0,$$
$$H_1: \quad \mu \neq \mu_0.$$

Finally, suppose that the sample size n is 25, and consider a test procedure such that H_0 is to be rejected if $|\overline{X}_n - \mu_0| \geq c$. Determine the value of c such that the size of the test will be 0.05.

11. Assume that $X_1, \ldots, X_9$ are i.i.d. having the Bernoulli distribution with parameter p. Suppose that we wish to test the hypotheses

$$H_0: \quad p = 0.4,$$
$$H_1: \quad p \neq 0.4.$$

Let $Y = \sum_{i=1}^{9} X_i$.

a. Find c_1 and c_2 such that

$$\Pr(Y \le c_1 | p = 0.4) + \Pr(Y \ge c_2 | p = 0.4)$$

is as close as possible to 0.1 without being larger than 0.1.

b. Let δ be the test that rejects H_0 if either $Y \le c_1$ or $Y \ge c_2$. What is the size of the test δ_c?

c. Draw a graph of the power function of δ_c.

13. Let X have the Poisson distribution with mean θ. Suppose that we wish to test the hypotheses

$$H_0: \quad \theta \le 1.0,$$
$$H_1: \quad \theta > 1.0.$$

Let δ_c be the test that rejects H_0 if $X \ge c$. Find c to make the size of δ_c as close as possible to 0.1 without being larger than 0.1.