7.5 Maximum Likelihood Estimators

2. It is not known what proportion p of the purchases of a certain brand of breakfast cereal are made by women and what proportion are made by men. In a random sample of 70 purchases of this cereal, it was found that 58 were made by women and 12 were made by men. Find the M.L.E. of p.

4. Suppose that $X_1, \dots, X_n$ form a random sample from the Bernoulli distribution with parameter θ, which is unknown, but it is known that θ lies in the open interval $0 < \theta < 1$. Show that the M.L.E. of θ does not exist if every observed value is 0 or if every observed value is 1.

6. Suppose that $X_1, \dots, X_n$ form a random sample from a normal distribution for which the mean μ is known, but the variance σ^2 is unknown. Find the M.L.E. of σ^2.

8.1 The Sampling Distribution of a Statistic

2. Suppose that a random sample is to be taken from the normal distribution with unknown mean θ and standard deviation 2. How large a random sample must be taken in order that $E_\theta(|\overline{X}_n - \theta|^2) \le 0.1$ for every possible value of θ?

4. For the conditions of Exercise 2, how large a random sample must be taken in order that $\Pr(|\overline{X}_n - \theta| \le 0.1) \ge 0.95$ for every possible value of θ?

8.2 The Chi-Square Distributions

1. Suppose that we will sample 20 chunks of cheese in Example 8.2.3. Let $T = \sum_{i=1}^{20}(X_i - \mu)^2/20$, where X_i is the concentration of lactic acid in the ith chunk. Assume that $\sigma^2 = 0.09$. What number c satisfies $\Pr(T \le c) = 0.9$?

4. Suppose that a point (X, Y) is to be chosen at random in the xy-plane, where X and Y are independent random variables and each has the standard normal distribution. If a circle is drawn in the xy-plane with its center at the origin, what is the radius of the smallest circle that can be chosen in order for there to be probability 0.99 that the point (X, Y) will lie inside the circle?

9. Suppose that $X_1, \dots, X_n$ form a random sample from the normal distribution with mean μ and variance σ^2. Find the distribution of

$$\frac{n(\overline{X}_n - \mu)^2}{\sigma^2}.$$

8.3 Joint Distribution of the Sample Mean and Sample Variance

1. Assume that $X_1, \dots, X_n$ form a random sample from the normal distribution with mean μ and variance σ^2. Show that $\widehat{\sigma^2}$ has the gamma distribution with parameters $(n-1)/2$ and $n/(2\sigma^2)$.

7. Suppose that $X_1, \dots, X_n$ form a random sample from the normal distribution with mean μ and variance σ^2, and let $\widehat{\sigma^2}$ denote the sample variance. Determine the smallest values of n for which the following relations are satisfied:

a. $\Pr\left(\frac{\widehat{\sigma^2}}{\sigma^2} \le 1.5\right) \ge 0.95$

b. $\Pr\left(|\widehat{\sigma^2} - \sigma^2| \le \frac{1}{2}\sigma^2\right) \ge 0.8$

8. Suppose that X has the χ^2 distribution with 200 degrees of freedom. Explain why the central limit theorem can be used to determine the approximate value of $\Pr(160 < X < 240)$ and find this approximate value.