6.2 The Law of Large Numbers

2. Suppose that X is a random variable for which

$$\Pr(X \geq 0) = 1 \text{ and } \Pr(X \geq 10) = 1/5.$$

Prove that $E(X) \geq 2$.

6. Suppose that $X_1, \ldots, X_n$ form a random sample of size n from a distribution for which the mean is 6.5 and the variance is 4. Determine how large the value of n must be in order for the following relation to be satisfied:

$$\Pr(6 \leq \overline{X}_n \leq 7) \geq 0.8.$$

8. Suppose that 30 percent of the items in a large manufactured lot are of poor quality. Suppose also that a random sample of n items is to be taken from the lot, and let Q_n denote the proportion of the items in the sample that are of poor quality. Find a value of n such that $\Pr(0.2 \leq Q_n \leq 0.4) \geq 0.75$ by using **(a)** the Chebyshev inequality and **(b)** the tables of the binomial distribution at the end of this book.

6.3 The Central Limit Theorem

3. Suppose that the distribution of the number of defects on any given bolt of cloth is the Poisson distribution with mean 5, and the number of defects on each bolt is counted for a random sample of 125 bolts. Determine the probability that the average number of defects per bolt in the sample will be less than 5.5.

10. A random sample of n items is to be taken from a distribution with mean μ and standard deviation σ.

a. Use the Chebyshev inequality to determine the smallest number of items n that must be taken in order to satisfy the following relation:

$$\Pr\left(|\overline{X}_n - \mu| \leq \frac{\sigma}{4}\right) \geq 0.99.$$

b. Use the central limit theorem to determine the smallest number of items n that must be taken in order to satisfy the relation in part (a) approximately.

13. Suppose that $X_1, \ldots, X_n$ form a random sample from a normal distribution with unknown mean θ and variance σ^2. Assuming that $\theta \neq 0$, determine the asymptotic distribution of $\overline{X}_n^3$.