Parametric Statistics

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What is probability?

Probability

is a language for quantifying uncertainty. It is a way to quantify how likely something is to occur.

Experiment

An experiment is any real or hypothetical process, in which the **possible outcomes** can be identified ahead of time. Events are **sets** of possible outcomes. Probability is then a way to describe how likely each event is.

Possible experiments

- We toss a coin 2 times.
  Possible Outcomes:
  Sample Space:
  Examples of Events:
  Probability of each event:
- We measure the temperature.
  Possible Outcomes:
  Sample Space:
  Examples of events:
  Probability of each event:

Sample Spaces

- The sample space is Ω is the set of possible outcomes of an experiment.
- $\omega \in \Omega$ is are called **sample outcomes**, or **elements**.
- Subsets of Ω are called **events**.

Example:

Coin tossing: If you toss a coin twice then

$$\Omega = \{HH, HT, TH, TT\}$$

The event that both tosses are heads are:
The event that the first toss is heads is:

Sample Space: Examples

Let ω be the outcome of measuring temperature. A sample space for this experiment is $\Omega = (-\infty, \infty)$. Is this accurate?

- What are the elements of Ω?
- Example events: temperature is 15.5.
- Example events: temperature is at least 10 but lower than 20 is $A = [10, 20)$.

Probability Measure

We want to assign a real number $\mathbb{P}(A)$ to every event A which represents how likely event A is to occur. This is called the probability of A.

- Axiom 1: $\mathbb{P}(A) \geq 0$ for every A
- Axiom 2: $\mathbb{P}(\Omega) = 1$
- Axiom 3: for an infinite sequence $A_1, A_2, \ldots$ of disjoint events

$$\mathbb{P}(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} \mathbb{P}(A_i)$$

Properties of Probabilities (1)

Based on the axioms of probability, we can derive several properties:

- The probability of an impossible event is 0:

$$P(\emptyset) = 0$$

- Axiom 3 also holds for finite sequences of events:

$$P(\bigcup_{i=1}^{N} A_i) = \sum_{i=1}^{N} P(A_i)$$

The probability of any event is no more than 1:

$$P(A) \leq 1$$

Properties of Probabilities (2)

The law of total probability

Let $B_1, \dots, B_n$ be a partition of the sample space. Then for any event A,

$$P(A) = \sum_i P(A \cap B_i)$$

Let's prove it for a very simple partition: B, B^c. Reminder:

- Set partitioning: $A = (A \cap B) \cup (A \cap B^c)$.
- Distribution law: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
- $(A \cap B), (A \cap B^c)$ are disjoint.

Properties of Probabilities (3)

- For any events A and B,

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$$

Example.

Two coin tosses. Let H_1 be the event that heads occurs on toss 1 and let H_2 be the event that heads occurs on toss 2. If all outcomes are equally likely, what is the $\mathbb{P}(H_1 \cup H_2)$?

Practice Exercises

Section	Exercises
1.4	1, 6, 8
1.5	3, 4, 8, 10, 14