Recitation 4

1. If X has the uniform distribution on the interval $[a, b]$, what is the value of the fifth central moment of X ?

2. If X has the uniform distribution on the interval $[a, b]$, write a formula for every even central moment of X.

3. Suppose that 20 percent of the students who took a certain test were from school A and that the arithmetic average of their scores on the test was 80 . Suppose also that 30 percent of the students were from school B and that the arithmetic average of their scores was 76. Suppose, finally, that the other 50 percent of the students were from school C and that the arithmetic average of their scores was 84 . If a student is selected at random from the entire group that took the test, what is the expected value of her score?

4. Suppose that a person's score X on a mathematics aptitude test is a number in the interval $(0, 1)$ and that his score Y on a music aptitude test is also a number in the interval $(0, 1)$. Suppose also that in the population of all college students in the United States, the scores X and Y are distributed in accordance with the following joint p.d.f.:

$$f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y) & \text{for } 0 \le x \le 1 \text{ and } 0 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) If a college student is selected at random, what predicted value of his score on the music test has the smallest M.S.E.?

   (b) What predicted value of his score on the mathematics test has the smallest M.A.E.?

5. In the previous exercise, are the scores of college students on the mathematics test and the music test positively correlated, negatively correlated, or uncorrelated?

6. Three men A, B, and C shoot at a target. Suppose that A shoots three times and the probability that he will hit the target on any given shot is $1/8$, B shoots five times and the probability that he will hit the target on any given shot is $1/4$, and C shoots twice and the probability that he will hit the target on any given shot is $1/2$. What is the expected number of times that the target will be hit?

7. Suppose that the random variables $X_1, \ldots, X_n$ form n Bernoulli trials with parameter p. Determine the conditional probability that $X_1 = 1$, given that

$$\sum_{i=1}^{n} X_i = k \quad (k = 1, \ldots, n)$$

8. In a clinical trial with two treatment groups, the probability of success in one treatment group is 0.5 , and the probability of success in the other is 0.6 . Suppose that there are five patients in each group. Assume that the outcomes of all patients are independent. Calculate the probability that the first group will have at least as many successes as the second group.

9. Suppose that in a certain book there are on the average λ misprints per page and that misprints occurred according to a Poisson process. What is the probability that a particular page will contain no misprints?

10. Suppose that X_1 and X_2 are independent random variables and that X_i has the Poisson distribution with mean $\lambda_i (i = 1, 2)$. For each fixed value of $k (k = 1, 2, \ldots)$, determine the conditional distribution of X_1 given that $X_1 + X_2 = k$.

11. Suppose that X has the geometric distribution with parameter p. Show that for every nonnegative integer k, $\Pr(X \ge k) = (1 - p)^k$.

12. If the temperature in degrees Fahrenheit at a certain location is normally distributed with a mean of 68 degrees and a standard deviation of 4 degrees, what is the distribution of the temperature in degrees Celsius at the same location?

13. Find the 0.25 and 0.75 quantiles of the Fahrenheit temperature at the location mentioned in the previous exercise.

14. If a random sample of 25 observations is taken from the normal distribution with mean μ and standard deviation 2 , what is the probability that the sample mean will lie within one unit of μ ?

15. Suppose that a random sample of size n is to be taken from the normal distribution with mean μ and standard deviation 2. Determine the smallest value of n such that

$$\Pr\left(\left|\bar{X}_n - \mu\right| < 0.1\right) \geq 0.9$$

16. Suppose that n items are being tested simultaneously, the items are independent, and the length of life of each item has the exponential distribution with parameter β. Determine the expected length of time until three items have failed.

17. Suppose that a certain examination is to be taken by five students independently of one another, and the number of minutes required by any particular student to complete the examination has the exponential distribution for which the mean is 80 . Suppose that the examination begins at 9:00 A.M. Determine the probability that at least one of the students will complete the examination before 9:40 A.M.