Recitation 2

1. Suppose that a random variable X has a discrete distribution with the following p.f.:

$$f(x) = \begin{cases} \frac{c}{2^x} & \text{for } x = 0, 1, 2, \ldots, \\ 0 & \text{otherwise.} \end{cases}$$

   Find the value of the constant c.

2. A civil engineer is studying a left-turn lane that is long enough to hold seven cars. Let X be the number of cars in the lane at the end of a randomly chosen red light. The engineer believes that the probability that $X = x$ is proportional to $(x + 1)(8 - x)$ for $x = 0, \ldots, 7$ (the possible values of X).
   (a) Find the p.f. of X.
   (b) Find the probability that X will be at least 5.

3. Suppose that the p.d.f. of a random variable X is as follows:

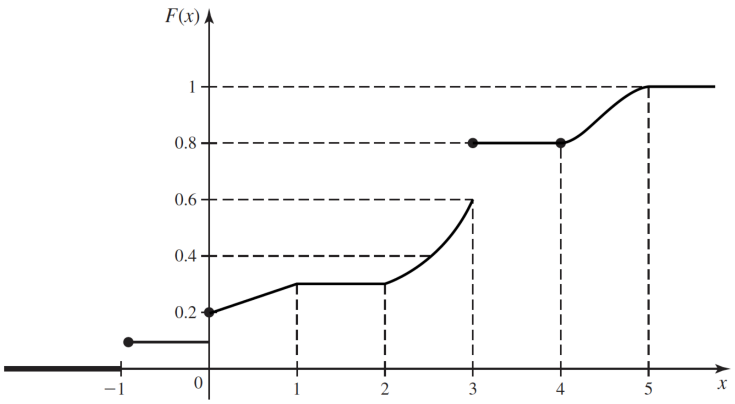
$$f(x) = \begin{cases} cx^2 & \text{for } 1 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the value of the constant c and sketch the p.d.f.
   (b) Find the value of $\Pr(X > 3/2)$.

4. Suppose that the p.d.f. of a random variable X is as follows:

$$f(x) = \begin{cases} ce^{-2x} & \text{for } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

5. Suppose that the c.d.f. F of a random variable X is as sketched below. Find each of the following probabilities:



a. $\Pr(X = -1)$
b. $\Pr(X < 0)$
c. $\Pr(X \le 0)$
d. $\Pr(X = 1)$
e. $\Pr(0 < X \le 3)$
f. $\Pr(0 < X < 3)$
g. $\Pr(0 \le X \le 3)$
h. $\Pr(1 < X \le 2)$
i. $\Pr(1 \le X \le 2)$
j. $\Pr(X > 5)$
k. $\Pr(X \ge 5)$
I. $\Pr(3 \le X \le 4)$

6. Suppose that the c.d.f. of a random variable X is as follows:

$$F(x) = \begin{cases} e^{x-3} & \text{for } x \le 3, \\ 1 & \text{for } x > 3. \end{cases}$$

   Find and sketch the p.d.f. of X.

7. Suppose that X has the p.d.f.

$$f(x) = \begin{cases} 2x & \text{if } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

   Find and sketch the c.d.f. or X.