Recitation 1

1. If 50 percent of the families in a certain city subscribe to the morning newspaper, 65 percent of the families subscribe to the afternoon newspaper, and 85 percent of the families subscribe to at least one of the two newspapers, what percentage of the families subscribe to both newspapers?

2. For two arbitrary events A and B, prove that

$$\Pr(A) = \Pr(A \cap B) + \Pr\left(A \cap B^c\right).$$

3. A point (x, y) is to be selected from the square S containing all points (x, y) such that $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Suppose that the probability that the selected point will belong to each specified subset of S is equal to the area of that subset. Find the probability of each of the following subsets: (a) the subset of points such that $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 \geq \frac{1}{4}$; (b) the subset of points such that $\frac{1}{2} < x + y < \frac{3}{2}$; (c) the subset of points such that $y \leq 1 - x^2$; (d) the subset of points such that $x = y$.

4. Let the sample space S be the unit cube, i.e. $0 < x < 1, 0 < y < 1$ and $0 < z < 1$. For $A \subset S$ we define $P(A) = \text{Volume}(A)$.
   (a) Show that P is a probability function.
   (b) Find the probability of $A = \{(x, y, z) : 0 < x < y < 1 \text{ and } z \leq y \exp(-x)\}$. Hint: You find the volume of A by integrating $y \exp(-x)$ over the right values of x and y.

5. Which of the following two numbers is larger: $\begin{pmatrix} 93 \\ 30 \end{pmatrix}$ or $\begin{pmatrix} 93 \\ 31 \end{pmatrix}$?

6. Which of the following two numbers is larger: $\begin{pmatrix} 93 \\ 30 \end{pmatrix}$ or $\begin{pmatrix} 93 \\ 63 \end{pmatrix}$?

7. A box contains 24 light bulbs, of which four are defective. If a person selects four bulbs from the box at random, without replacement, what is the probability that all four bulbs will be defective?

8. If six dice are rolled, what is the probability that each of the six different numbers will appear exactly once?

9. Each time a shopper purchases a tube of toothpaste, he chooses either brand A or brand B. Suppose that for each purchase after the first, the probability is 1/3 that he will choose the same brand that he chose on his preceding purchase and the probability is 2/3 that he will switch brands. If he is equally likely to choose either brand A or brand B on his first purchase, what is the probability that both his first and second purchases will be brand A and both his third and fourth purchases will be brand B?

10. A machine produces defective parts with three different probabilities depending on its state of repair. If the machine is in good working order, it produces defective parts with probability 0.02 . If it is wearing down, it produces defective parts with probability 0.1. If it needs maintenance, it produces defective parts with probability 0.3 . The probability that the machine is in good working order is 0.8 , the probability that it is wearing down is 0.1 , and the probability that it needs maintenance is 0.1 . Compute the probability that a randomly selected part will be defective.

11. Suppose that a person rolls two balanced dice three times in succession. Determine the probability that on each of the three rolls, the sum of the two numbers that appear will be 7 .

12. Suppose that A, B, and C are three independent events such that $\Pr(A) = 1/4, \Pr(B) = 1/3$, and $\Pr(C) = 1/2$. (a) Determine the probability that none of these three events will occur. (b) Determine the probability that exactly one of these three events will occur.

13. Suppose that the probability that any particle emitted by a radioactive material will penetrate a certain shield is 0.01 . If 10 particles are emitted, what is the probability that exactly one of the particles will penetrate the shield?

14. Consider again the conditions of the previous exercise. If 10 particles are emitted, what is the probability that at least one of the particles will penetrate the shield?

15. In a certain city, 30 percent of the people are Conservatives, 50 percent are Liberals, and 20 percent are Independents. Records show that in a particular election, 65 percent of the Conservatives voted, 82 percent of the Liberals voted, and 50 percent of the Independents voted. If a person in the city is selected at random and it is learned that she did not vote in the last election, what is the probability that she is a Liberal?

16. Suppose that when a machine is adjusted properly, 50 percent of the items produced by it are of high quality and the other 50 percent are of medium quality. Suppose, however, that the machine is improperly adjusted during 10 percent of the time and that, under these conditions, 25 percent of the items produced by it are of high quality and 75 percent are of medium quality.
    (a) Suppose that five items produced by the machine at a certain time are selected at random and inspected. If four of these items are of high quality and one item is of medium quality, what is the probability that the machine was adjusted properly at that time?
    (b) Suppose that one additional item, which was produced by the machine at the same time as the other five items, is selected and found to be of medium quality. What is the new posterior probability that the machine was adjusted properly?