Chapter 9: Hypothesis Testing

Sections

- 9.1 Problems of Testing Hypotheses

Introduction

Statistical Inference: Given a probability model $f(x|\theta)$ (and possibly a prior $p(\theta)$) we may be interested in

- Parameter estimation - Chapters 7 and 8
- Making decisions - Hypothesis testing, Chapter 9
  - E.g. If the disease affects 2% or more of the population, the state will launch a costly public health campaign.
    Do we have evidence that θ is higher than 2% ?
- Other things like, prediction, experimental design, etc.

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Some of what lies ahead:

- The concept of testing hypotheses and several tools and notation involving that
- "Famous" tests, such as t-Test, two-sample t-Test, F-Test

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- Customers are expected to use their own envelopes and stamps to return their payments
- Currently, the time it takes to pay bills has a mean of 24 days and a standard deviation of 6 days.

Example: After-School, continued

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- She assumes that the time to pay a bill follows the normal distribution $N(\mu, \sigma^2)$, both parameters unknown
- Using the data from her experiment, how can she conclude whether this plan will be profitable?

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- *Hypothesis testing*: Inferential method to decide between two complimentary *hypotheses* about a parameter.

$$H_0 : \theta \in \Omega_0 \qquad \text{Null Hypothesis}$$
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After-School example: May be interested in testing the following:

$$H_0 : \mu \geq 22$$
$$H_1 : \mu < 22$$

Simple vs Composite, One or Two sided

Simple and Composite hypotheses:

- If Ω_i contains only a single value, $\Omega_i = \{\theta_i\}$, then H_i is a *simple hypothesis*
- If Ω_i contains more than a single value then H_i is a *composite hypothesis*

One-Sided and Two-Sided (for a one-dimensional θ)

- *One-sided* hypotheses:

$$\begin{array}{ll} H_0: & \theta \geq \theta_0 \\ H_1: & \theta < \theta_0 \end{array} \quad \text{or} \quad \begin{array}{ll} H_0: & \theta \leq \theta_0 \\ H_1: & \theta > \theta_0 \end{array}$$

- If the Null Hypothesis is simple the alternative is usually *Two-sided*:

$$H_1: \ \theta \neq \theta_0$$

Test procedure in general

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Test procedure based on a statistic

- Let $T = r(\mathbf{X})$ be a statistic and $R \subset \mathbb{R}$
- Procedure: Reject H_0 if $T \in R$, do not reject H_0 if $T \notin R$
- T is called a *test statistic* and R is called the *rejection region* of the test

Example: After-School

We had $X_1, \ldots, X_n$ i.i.d. $N(\mu, \sigma^2)$ and we are interested in testing

$$H_0 : \mu \geq 22 \qquad \text{and} \qquad H_1 : \mu < 22$$

- It seems reasonable to reject H_0 if $\overline{X}_n$ is much lower than 22.
- The test procedure would be to reject H_0 if $\overline{X}_n < 22 - c$ for some positive constant c
- What is the critical region for this test?

It is usually rather difficult to work with a critical region.
We usually use a test statistic and a rejection region.

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- We could use the test statistic

$$U = \frac{\sqrt{n}(\overline{X}_n - 22)}{\sigma'}$$

  and reject H_0 if U is very small, say less than some constant c.
- What is the rejection region for this test ?

Power function

We can describe the properties of a test procedure by:

Power function

Let δ be a test procedure. The *power function* $\pi(\theta|\delta)$ (or just $\pi(\theta)$) is the probability of rejecting H_0 for the given θ:

$$\pi(\theta|\delta) = P(\mathbf{X} \in S_1|\theta) \quad \text{for } \theta \in \Omega$$

If we use a test statistic T we can write

$$\pi(\theta|\delta) = P(T \in R|\theta) \quad \text{for } \theta \in \Omega$$

- Ideal power function:

$$\begin{aligned} \pi(\theta|\delta) &= 1 \qquad \text{for } \theta \in \Omega_1 \\ \pi(\theta|\delta) &= 0 \qquad \text{for } \theta \in \Omega_0 \end{aligned}$$

Power function for after-school

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Consider the following test procedure:

- Assume that σ^2 is known, $\sigma^2 = 6^2$
- Reject H_0 if $\overline{X}_n < 22 - c$.
  - $\overline{X}_n$ is our test statistic
  - The rejection region is $(-\infty, 22 - c)$
- The power function is

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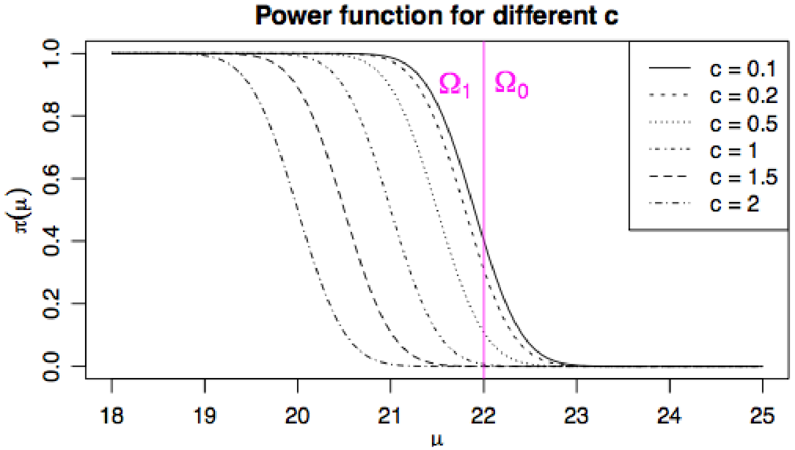
$$\pi(\theta|\delta) = \Phi\left(\frac{22 - c - \mu}{6/\sqrt{220}}\right)$$

We can then pick c that gives us the best power function

Power function for after-school



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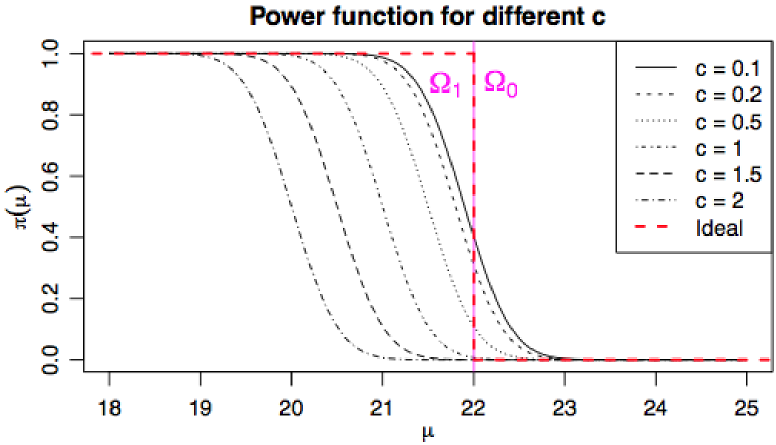

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Relation to power function:

- If $\theta \in \Omega_0$: $\pi(\theta|\delta) =$ probability of type I error
- If $\theta \in \Omega_1$: $1 - \pi(\theta|\delta) =$ probability of type II error

Level and size of tests

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- The *size* $\alpha(\delta)$ of a test is defined as

$$\alpha(\delta) = \sup_{\theta \in \Omega_0} \pi(\theta|\delta)$$

Level and size of tests

Corollary 9.1.1: Level and size

A test δ is a level α_0 test if and only if $\alpha(\delta) \leq \alpha_0$

When the null hypothesis is simple ($H_0 : \theta = \theta_0$) then $\alpha(\delta) = \pi(\theta_0|\delta)$

Example:

- Consider again the After-School example
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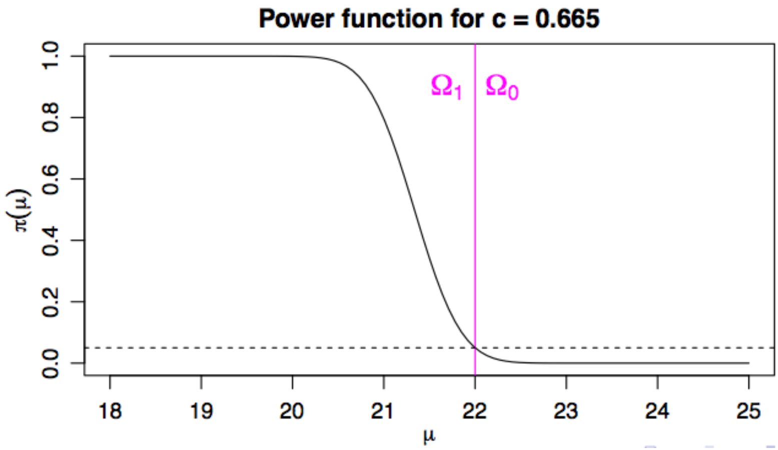
- Pick a c so that the test has level 0.05
- What is the size of test?

Power function for a size 0.05 test for after-school



Choosing the Null and alternative hypotheses

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- The type II error here (not rejecting H_0 when it is not true) represents lost opportunity of profit but at least the district will not lose (more) money.

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- We usually arrange the Null and Alternative hypotheses so that the type I error is the more serious one.
- Another way to think about it: The statement we are trying to "prove" we set as the alternative hypothesis

Bernoulli example: Prevalence of a disease

Say we are interested in the prevalence of some disease

- If the disease affects 2% or more of the population, the state will launch a costly public health campaign.

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Which hypothesis do we test?

a) $H_0 : p \leq 0.02$ and $H_1 : p > 0.02$

b) $H_0 : p \geq 0.02$ and $H_1 : p < 0.02$

Bernoulli example: Prevalence of a disease

Some issues to ponder:

- If we wrongly reject $p \leq 0.02$:
  We think that the disease is more common than it is and launch a costly public health campaign that is not as needed as we thought

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- Which error is worse? Put that statement in the Null Hypothesis

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c	1	2	3	4	5	6
$P(Y > c \mid p = 0.02)$	0.477	0.216	0.077	0.022	0.005	0.001

E.g. in R: `1-pbinom(c, size=80, prob=0.02)`

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