Lecture Summary

Sample Mean

Definition (Sample mean)

Let $X_1, \ldots X_n$ be random variables. Their average

$$\overline{X}_n = \frac{X_1 + \cdots + X_n}{n}$$

is called their *sample mean*.

- What happens when $n \to \infty$?

Properties of the sample mean

Theorem (Mean and variance of the sample mean)

Let $X_1, \dots X_n$ be a random sample from a distribution with mean μ and variance σ^2. Then $E(\overline{X}_n) = \mu$, and $Var(\overline{X}_n) = \sigma^2/n$.

Inequalities

Theorem (Markov Inequality)

Let X be a random variable such that $P(X \geq 0) = 1$. Then for every real number t,

$$P(X \geq t) \leq \frac{E(X)}{t}.$$

Theorem (Chebysev Inequality)

Let X be a random variable for which $Var(X)$ exists. Then for every number $t > 0$,

$$P(|X - E(X)| \geq t) \leq \frac{Var(X)}{t^2}.$$

Convergence in probability

Arithmetic convergence	Convergence in probability
Series S_n of numbers converges to number ℓ	Series X_n of random variables converges to number α
$lim_{n\to\infty} S_n = \ell$ or $S_n \to \ell$	$X_n \xrightarrow{p} \alpha$
S_n gets arbitrarily close to ℓ	The probability distribution of X gets more and more concentrated around α
$\forall \epsilon > 0,\ \exists n_0 : \forall n > n_0 \lvert S_n - \ell \rvert < \epsilon$	$\forall \epsilon > 0,$ $lim_{n\to\infty} P(\lvert X_n - \alpha \rvert < \epsilon) = 1$

Weak Law of Large Numbers

Theorem (Weak Law of Large Numbers)

Suppose that $X_1, \ldots, X_n$ form a random sample from a distribution (i.e., $X_i, \ldots, X_n$ are i.i.d.) for which the mean is μ and the variance is finite. Let $\overline{X_n}$ denote the sample mean. Then

$$\overline{X_n} \xrightarrow{p} \mu.$$

Theorem (Continuous Functions of Random Variables)

. If $\overline{Z_n} \xrightarrow{p} b$, and if $g(z)$ is a function that is continuous at $z = b$, then $g(Z_n) \xrightarrow{p} g(b)$

Example

- You are doing a poll on "ratio of people who believe it was a proper penalty".
- True ratio: p, estimate $\overline{X_n}$
- No guarantee for finding exactly p, so

$$P(|\bar{X}_n - p| \geq 0.01) \leq 0.05$$

- Apply Chebysev inequality with $t = 0.01$: n = 50,000.
- Apply CLT: ?