Lecture Summary

Joint/Conditional/Marginal PDFs

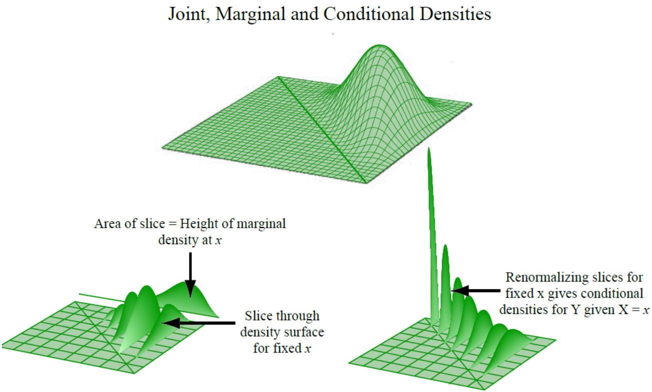

Image by MIT OpenCourseWare, adapted from *Probability*, by J. Pittman, 1999.

Joint/Conditional/Marginal PDFs

Example

Suppose that the joint pdf of X and Y is

$$f(x,y) = \begin{cases} \frac{21}{4}x^2y, & x^2 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the marginal pdfs $f_X(x), f_Y(y)$.

Conditional Expectation

- Conditional distributions are distributions, so they have expectations and variances.
- $E(Y|X = x) = \sum_y yP(y|X = x)$ is the conditional expectation of Y if you know $X = x$ (a number).
- $E(Y|X) = h(X)$ is a function of X. For every possible value x of X, $E(Y|X)$ takes the value $E(Y|X = x)$. So $E(Y|X)$ is a random variable.
- Law of total expectation/Law of iterated expectations:

$$E[E(Y|X)] = E(Y)$$

Conditional Variance

- $Var(Y|X=x) = E[(Y - E(Y|X=x))^2|X=x]$ is the variance of Y given $X = x$ (a number).
- $Var(Y|X)$ is a function of Y that takes the value $Var(Y|X=x)$ for every possible value x of X. So $Var(Y|X)$ is a random variable.
- Law of total variance:

$$Var(Y) = E[Var(Y|X)] + Var(E(Y|X))$$

Practice Exercises

3.4	4
3.5	7,8
3.6	2, 12
4.7	7