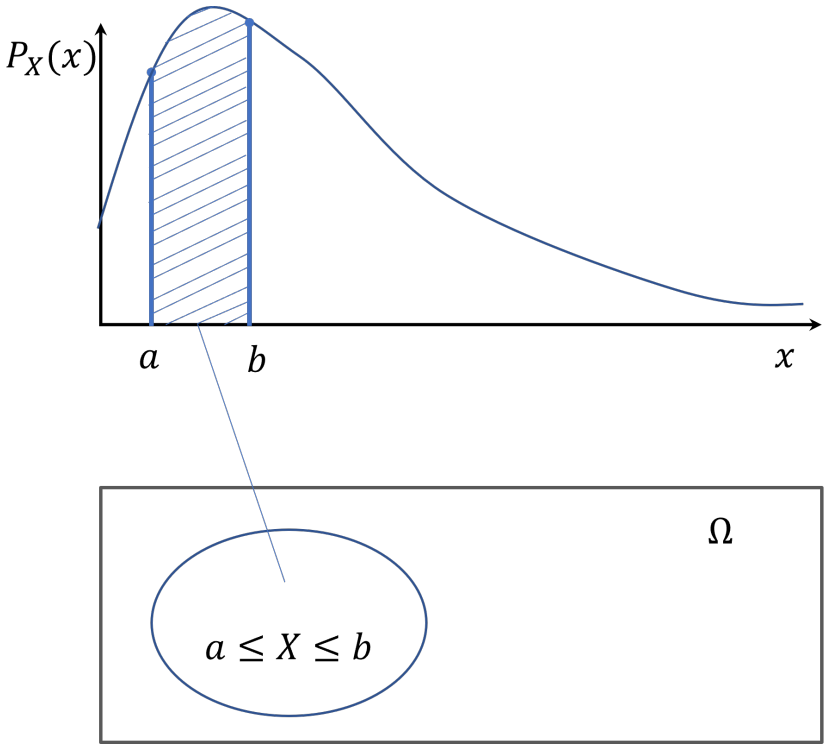
Lecture Summary

3.2 Continuous Distributions

3.3 The CDF

Continuous Random Variables

What if an r.v. takes values in a continuous range?



- $P(a \leq X \leq b) = \int_a^b f_X(x)dx$
- $P(x \leq X \leq x + \delta) = f_x(x)\delta$
- $\int_{-\infty}^{\infty} f_X(x) = 1$
- Density is not probability! We can integrate to compute probabilities.

Cumulative Distribution Function

Definition

The cumulative distribution function (CDF) is

$$F_X(x) = P(X \leq x) = \begin{cases} \int_{-\infty}^{x} f_X(t)dt, & \text{if } X \text{ is continuous} \\ \sum_{k \leq x} p_X(x) & \text{if } X \text{ is discrete} \end{cases}$$

- The CDF is non-decreasing as X increases.
- $lim_{x \to -\infty} F_X(x) = 0$, $lim_{x \to \infty} F_X(x) = 1$.
- The CDF is continuous from the right.
- $P(a < X \leq b) = F_X(b) - F_X(a)$.
- The CDF is well-defined for mixed variables.
- The pdf is the derivative of the CDF (where the derivative exists).

Quantiles

Definition (The Quantile Function)

Let X be a random variable with CDF F. For each p strictly between 0 and 1, define $F^-1(p)$ to be the smallest value x for which $F(X) \geq p$. Then $F^-1(p)$ is called the p-quantile of X or the 100p percentile of X. The function F^1 defined on (0,1) is called the *quantile function.*

- Example: Compute the quantile function for the uniform distribution.
- Example: Standardized test scores.

Identically Distributed

Definition

Identically Distributed Random variables X and Y are identically distributed (id) if for every set A we have $P(X \in A) = P(Y \in A)$

- Note:X and Y are NOT necessarily the same.
- Example: Let X and Y be the number of head and tails, respectively, in n tosses of a fair coin. They are not the same random variable, but they have the same distribution!

Expectation

We can extend this to multiple random variables.

Expectation of a continuous random variable

$$E(X) = \int_x x f(x) dx$$

LOTUS for continuous random variables

$$E(g(X) = \int_x g(x) f_X(x) dx$$

Practice Exercises

Section	Exercises
3.2	4, 8
3.3	4, 6, 15