# Lecture Summary

- Sample mean and variance
- Markov and Chebysev Inequalities
- Convergence of Random Variables
- Weak law of large numbers

Material can be found in Chapter 6 of Degroot and Schervish.

# Sample Mean

### Definition (Sample mean)

Let $X_1, \ldots X_n$ be random variables. Their average

$$\overline{X}_n = \frac{X_1 + \cdots + X_n}{n}$$

is called their *sample mean*.

- What happens when $n \to \infty$?

# Properties of the sample mean

**Theorem (Mean and variance of the sample mean)**

*Let $X_1, \ldots X_n$ be a random sample from a distribution with mean $\mu$ and variance $\sigma^2$. Then $E(\overline{X}_n) = \mu$, and $Var(\overline{X}_n) = \sigma^2/n$.*

# Inequalities

### Theorem (Markov Inequality)

*Let $X$ be a random variable such that $P(X \geq 0) = 1$. Then for every real number $t$,*

$$P(X \geq t) \leq \frac{E(X)}{t}.$$

### Theorem (Chebysev Inequality)

*Let $X$ be a random variable for which $Var(X)$ exists. Then for every number $t$,*

$$P(|X - E(X)| \geq t) \leq \frac{Var(X)}{t^2}.$$

# Convergence in probability

| Arithmetic convergence | Convergence in probability |
| --- | --- |
| Series $S_n$ of numbers converges to number $\ell$ | Series $X_n$ of random variables converges to number $\alpha$ |
| $lim_{n \to \infty} S_n = \ell$ or $S_n \to \ell$ | $X_n \underset{p}{\to} \alpha$ |
| $S_n$ gets arbitrarily close to $\ell$ | The probability distribution of $X$ gets more and more concentrated around $\alpha$ |
| $\forall \epsilon > 0 \; \exists n_0 : \forall n > n_0 \lvert S_n - \ell \rvert < \epsilon$ | $\forall \epsilon > 0, lim_{n \to \infty} P(\lvert X_n - \alpha \rvert < \epsilon) = 1$ |

# Properties of Convergence in probability

# Weak Law of Large Numbers

## Theorem (Weak Law of Large Numbers)

*Suppose that $X_1, \ldots, X_n$ form a random sample from a distribution (i.e., $X_i, \ldots, X_n$ are i.i.d.) for which the mean is $\mu$ and the variance is finite. Let $\overline{X_n}$ denote the sample mean. Then*

$$\overline{X_n} \xrightarrow{p} \mu.$$