Parametric Statistics

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Lecture Summary

- Discrete Random Variables.
- The probability mass function.
- Distributions: Bernoulli, Binomial, Geometric.

Material can be found in Chapter 3 of Degroot and Schervish.

Random Variables

Random Variable

A random variable is a mapping $X : \Omega \rightarrow \mathbb{R}$ that assigns a real number $X(\omega)$ to each outcome ω.

Example

Consider the experiment of flipping a coin 10 times, and let $X(\omega)$ denote the number of heads in the outcome ω. For example, if $\omega = HHHTHHHTTHT$, $X(\omega) = 6$.

Random Variables

- Why do we need random variables? (easier to work with than original sample space)
- A random variable is NOT a variable (in the algebraic sense).
- A random variable takes a specific value AFTER the experiment is conducted.

Notation

- Letter near the end of the alphabet since it is a variable in the context of the experiment.
- Capital letter to distinguish from algebraic variable.
- Lower case denotes a specific value of the random variable.

Random Variables

- An assignment of a value (number) to every possible outcome.
- Strictly speaking: A function from the sample space Ω to the real numbers.
- It can take discrete or continuous values.
- We will start with discrete values to build our intuition, and then proceed to continuous.

Probability mass function

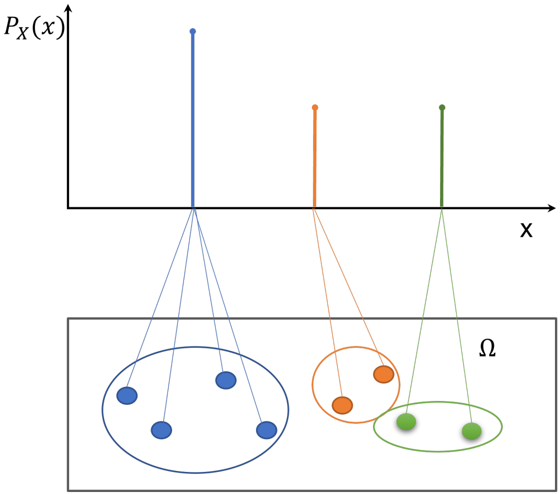
Definition (Probability (mass) function)

If a random variable X has a discrete distribution, the probability function of X is defined as the function p_X such that

$$P_X(x) = \mathbb{P}(X = x) = P(\omega \in \Omega \quad s.t. \quad X(\omega) = x).$$

The closure of the set $x : P_X(x) > 0$ is called the support of the (distribution of) X.

Computing Probability mass functions



Bernoulli distribution

Some distributions come up so often, that they have a name.

Definition

A random variable X that only takes two values 0 (failure) and 1 (success) with $P(X = 1) = p$ has the Bernoulli distribution with parameter p.

Example: You flip a coin one time. X is the result of the coin flip. You can define "Heads" as success and "Tails" as failure (or the other way around, it does not matter) Then X follows a Bernoulli distribution with parameter p.

$$P_X(x) = p^x(1-p)^{1-x}$$

Binomial Distribution

Now imagine you toss the fair coin 5 times. Some possible outcomes are 00000, 1000, 01100 etc. Let X be the number of successes (heads) after the 5 times. Then X follows a Binomial distribution with parameters (5, 0.5).

Definition

The binomial distribution with parameters n and p is the discrete probability distribution of the number of successes in a sequence of n independent experiments, each with a binary outcome: success (with probability p) or failure (with probability $q = 1 - p$). The pmf of the binomial distribution is

$$P_X(x) = \binom{n}{x} p^x (1-p)^{1-x} \text{ for } x = 0, \ldots, n,$$

Geometric distribution

Now imagine you toss the fair coin until you get heads. times. Some possible outcomes are 1, 01, 0001, $0 \ldots 1$, etc. Let X be the trial of the first success (heads). Then X follows a Geometric distribution parameter 0.5.

Definition