1. Random variables X and Y can take any value in the set $\{1, 2, 3\}$. We are given the following information about their joint PMF, where the entries indicated by a ? are left unspecified:

y	3	1/12	1/12	?
	2	2/12	?	?
	1	1/12	2/12	0
		1	2	3
			x	

   (a) What is $P_x(X = 1)$?

   (b) Compute and sketch the conditional pmf or Y given X=1.

   (c) Find the conditional expectation $E(Y|X = 1)$

   (d) Let B be the event that $X \leq 2, Y \leq 2$. If we know that X and Y are independent given B, what is $P(2, 2)$?

2. (4.2.7) Suppose that on each play of a certain game a gambler is equally likely to win or to lose. Suppose that when he wins, his fortune is doubled, and that when he loses, his fortune is cut in half. If he begins playing with a given fortune c, what is the expected value of his fortune after n independent plays of the game?

3. Suppose that the proportion of defective items in a large manufactured lot is 0.1. Use the central limit theorem to determine the smallest sample of items that must be taken from the lot in order for the probability to be at least 0.99 that the proportion of defective items in the sample will be less than 0.13?

4. A statistician wants to estimate the mean height h (in meters) of a population, based on n i.i.d. samples $X_1, \ldots, X_n$. He uses the sample mean $\overline{X}_n = (X_l + ... + X_n)/n$ as the estimate of h, and a rough guess of 1.0 meters for the standard deviation of the samples.

   (a) How large should n be so that the standard deviation of $\overline{X}_n$ is at most 1 centimeter?

   (b) How large should n be so the estimate is within 5 centimeters from h, with probability at least 0.99 (use the CLT)?

5. (7.2.2) Suppose that the proportion of defective items in a large manufactured lot is θ. Suppose also that when eight items are selected at random from the lot, it is found that exactly two of them are defective. Find the MLE estimate $\hat{\theta}_{MLE}$ for θ. Find the bias and the mean squared error of $\hat{\theta}_{MLE}$.

6. Suppose that a regular light bulb, a long-life light bulb, and an extra-long-life light bulb are being tested. The lifetime X_1 of the regular bulb has the exponential distribution with mean θ, the lifetime X_2 of the long-life bulb has the exponential distribution with mean 2θ, and the lifetime X_3 of the extra-long-life bulb has the exponential distribution with mean 3θ. *Reminder: The exponential distribution has density* $f(x) = \lambda e^{-\lambda x}, x > 0$ *and* $E(X) = \frac{1}{\lambda}$. Determine the M.L.E. of θ based on the observations X_1, X_2, X_3.

7. (7.5.2) It is not known what proportion p of the purchases of a certain brand of breakfast cereal are made by women and what proportion are made by men. In a random sample of 70 purchases of this cereal, it was found that 58 were made by women and 12 were made by men. Find the M.L.E. of p.

8. Suppose that against a certain opponent the number of points out local basketball team scores is normally distributed with unknown mean θ and unknown variance, σ^2. Suppose that over the course of the last 10 games between the two teams our team scored the following points: 59, 62, 59, 74, 70, 61, 62, 66, 62, 75.

   (a) Compute a 95% confidence interval for θ. Does 95% confidence mean that the probability θ is in the interval you just found is 95%?

(b) Now suppose that you learn that $\sigma^2 = 25$. Compute a 95% confidence interval for θ. How does this compare to the interval in (a)?

9. The volume in a set of wine bottles is known to follow a $N(\mu, 25)$ distribution. You take a sample of the bottles and measure their volumes. How many bottles do you have to sample to have a 95% confidence interval for μ with width at most 1?

Table 1: Part of the table of the Standard Normal CDF $\Phi(x) = \int_{-\infty}^{x} \frac{1}{(2\pi)^{1/2}} exp(-\frac{1}{2}u^2)du$.

z	0	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,9	0,81594	0,81859	0,82121	0,82381	0,82639	0,82894	0,83147	0,83398	0,83646	0,83891
1	0,84134	0,84375	0,84614	0,84849	0,85083	0,85314	0,85543	0,85769	0,85993	0,86214
1,1	0,86433	0,8665	0,86864	0,87076	0,87286	0,87493	0,87698	0,879	0,881	0,88298
1,2	0,88493	0,88686	0,88877	0,89065	0,89251	0,89435	0,89617	0,89796	0,89973	0,90147
1,3	0,9032	0,9049	0,90658	0,90824	0,90988	0,91149	0,91309	0,91466	0,91621	0,91774
1,4	0,91924	0,92073	0,9222	0,92364	0,92507	0,92647	0,92785	0,92922	0,93056	0,93189
1,5	0,93319	0,93448	0,93574	0,93699	0,93822	0,93943	0,94062	0,94179	0,94295	0,94408
1,6	0,9452	0,9463	0,94738	0,94845	0,9495	0,95053	0,95154	0,95254	0,95352	0,95449
1,7	0,95543	0,95637	0,95728	0,95818	0,95907	0,95994	0,9608	0,96164	0,96246	0,96327
1,8	0,96407	0,96485	0,96562	0,96638	0,96712	0,96784	0,96856	0,96926	0,96995	0,97062
1,9	0,97128	0,97193	0,97257	0,9732	0,97381	0,97441	0,975	0,97558	0,97615	0,9767
2	0,97725	0,97778	0,97831	0,97882	0,97932	0,97982	0,9803	0,98077	0,98124	0,98169
2,1	0,98214	0,98257	0,983	0,98341	0,98382	0,98422	0,98461	0,985	0,98537	0,98574
2,2	0,9861	0,98645	0,98679	0,98713	0,98745	0,98778	0,98809	0,9884	0,9887	0,98899
2,3	0,98928	0,98956	0,98983	0,9901	0,99036	0,99061	0,99086	0,99111	0,99134	0,99158
2,4	0,9918	0,99202	0,99224	0,99245	0,99266	0,99286	0,99305	0,99324	0,99343	0,99361
2,5	0,99379	0,99396	0,99413	0,9943	0,99446	0,99461	0,99477	0,99492	0,99506	0,9952
2,6	0,99534	0,99547	0,9956	0,99573	0,99585	0,99598	0,99609	0,99621	0,99632	0,99643
2,7	0,99653	0,99664	0,99674	0,99683	0,99693	0,99702	0,99711	0,9972	0,99728	0,99736

Table 2: Part of the table of the t Distribution. If X has a t distribution with m degrees of freedom, the table gives the value of x such that $P(X \leq x) = p$.

m	0.85	0.90	0.95	0.975	0.99	0.975
9	1.100	1.383	1.833	2.262	2.821	3.250
10	1.093	1.372	1.812	2.228	2.764	3.169
11	1.088	1.363	1.796	2.201	2.718	3.106
12	1.083	1.356	1.782	2.179	2.681	3.055