# Parametric Statistics

Sofia Triantafillou

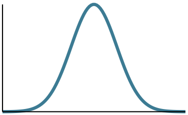
sof.triantafillou@gmail.com

University of Crete
Department of Mathematics and Applied Mathematics

# Summary

- The Normal Distribution (Chapter 5.6 in DGS).
- Joint/Conditional/Marginal PDFs (Chapters 3.4-3.9 in DGS).

# The normal distribution




Standard normal

$$\mathcal{N}(0,1) : f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

.

Normal with mean $\mu$ and variance $\sigma^2$

$$\mathcal{N}(0,1) : f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

.

**Theorem**

*If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $\alpha X + \beta \sim N(\alpha\mu + \beta, \alpha^2\sigma^2)$*

# Calculating probabilities with the Normal Distribution

- We want to estimate $P(X \leq a)$ when $X \sim \mathcal{N}(\mu, \sigma^2)$
- No closed form for $\int_{-\infty}^{a} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dt$
- If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $\frac{X-\mu}{\sigma} \sim \mathcal{N}(0, 1)$
- $P(X < a) = P(\frac{X-\mu}{\sigma} < \frac{a-\mu}{\sigma}) = \Phi(\frac{a-\mu}{\sigma})$.

# Joint/Conditional/Marginal PDFs

$$P(X, Y \in S) = \int \int_S f_{X,Y} dxdy$$

$$f_{X,Y}(x,y) \approx P(x \leq X \leq x + \delta, y \leq Y \leq y + \delta)\delta^2$$

$$\int_x \int y f_{X,Y}(x,y) = 1 \text{ (still a probability density function)}$$

Marginal Probability: $f_X(x) = \int_y f_{X,Y}(x,y)$ (integrate over all possible y)

# Joint/Conditional/Marginal PDFs

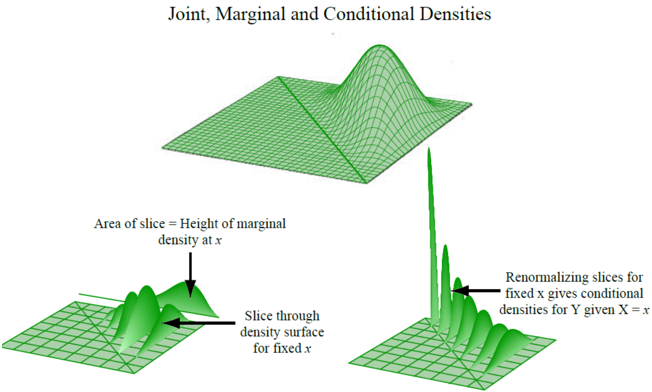



Image by MIT OpenCourseWare, adapted from *Probability*, by J. Pittman, 1999.

# Expectations

LOTUS for functions of multiple r.v.s:

$$E[g(X,Y)] = \sum_x \sum_y g(x,y) P_{X,Y}(x,y), \text{ discrete}$$

$$E[g(X,Y)] = \int_x \int_y g(x,y) f_{X,Y}(x,y) dxdy, \text{ continuous}$$

## Conditional Expectation

$$E(X|y) = \sum_x x P_{X|y} \text{ (for a given value } y \text{ of } Y)$$

$$E(X|Y) = \sum_x x P_{X|Y} \text{(for every value } y \text{ of } Y)$$

# Independent random variables

## Independent Discrete Random Variables

$$P_{X,Y}(x, y) = P_X(x)P_Y(y) \text{ for every pair } (x,\ y)$$

## Independent Continuous Random Variables

$$f_{X,Y}(x, y) = f_X(x)f_Y(y) \text{ for every pair } (x,\ y)$$

We can extend this to multiple random variables.

# Linearity of Variances for Independent Random Variables

Linearity of variances only holds for independent random variables.

**Theorem**

*Let $X_1, \ldots, X_n$ be a set of independent random variables. Then*

$$Var[X_1 + \cdots + X_n] = Var[X_1] + \cdots + Var[X_n]$$

Prove it for the case of two discrete variables.