Parametric Statistics

Probability/Random Variables/Expectation

Probability

1. Consider two events A and B with $P(A) = 0.4$ and $P(B) = 0.7$. Determine the maximum and the minimum possible values of $P(A \cap B)$, and explain why.

2. Suppose that 35 people are divided in a random manner into two teams in such a way that one team contains 10 people and the other team contains 25 people. What is the probability that two particular people A and B will be on the same team?

3. Suppose that when a machine is adjusted properly, 50 percent of the items produced by it are of high quality and the other 50 percent are of medium quality. Suppose, however, that the machine is improperly adjusted during 10 percent of the time and that, under these conditions, 25 percent of the items produced by it are of high quality and 75 percent are of medium quality.

   (a) Suppose that five items produced by the machine at a certain time are selected at random and inspected. If four of these items are of high quality and one item is of medium quality, what is the probability that the machine was adjusted properly at that time?

   (b) Suppose that one additional item, which was produced by the machine at the same time as the other five items, is selected and found to be of medium quality. What is the new probability that the machine was adjusted properly?

Random Variables

1. Suppose two balanced dice are rolled, and let X denote the absolute value of the difference between the two numbers that appear. Determine and sketch the pdf of X.

2. Suppose that the pdf of a random variable X is as follows:

$$f(x) = \begin{cases} cx^2, & \text{for } 1 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find the value of the constant c and sketch the pdf.

   (b) Find the value of $P(X > 3/2)$

3. Suppose that in an electric display sign there are three light bulbs in the first row and four light bulbs in the second row. Let X denote the number of bulbs in the first row that will be burned out at a specified time t , and let Y denote the number of bulbs in the second row that will be burned out at the same time t . Suppose that the joint p.m.f. of X and Y is as specified in the following table:

	Y				
X	0	1	2	3	4
0	0.08	0.07	0.06	0.01	0.01
1	0.06	0.10	0.12	0.05	0.02
2	0.05	0.06	0.09	0.04	0.03
3	0.02	0.03	0.03	0.03	0.04

   Determine each of the following probabilities:

   (a) $P(X = 2)$

   (b) $P(X \le 2, Y \le 2)$

   (c) $P(X > Y)$

   (d) $P(Y \ge 2)$

   (e) $P(X = Y)$

Expectation

1. Suppose that the proportion of defective items in a large lot is p, and suppose that a random sample of n items is selected from the lot. Let X denote the number of defective items in the sample, and let Y denote the number of non-defective items. Find $E(X - Y)$.

2. Suppose that the random variable X has the uniform distribution on the interval [0, 1], that the random variable Y has the uniform distribution on the interval [5, 9], and that X and Y are independent. Suppose also that a rectangle is to be constructed for which the lengths of two adjacent sides are X and Y . Determine the expected value of the area of the rectangle.

3. Suppose that one word is selected at random from the sentence

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   THE GIRL PUT ON HER BEAUTIFUL RED HAT.
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   If X denotes the number of letters in the selected word, what is the value of $E(X)$ and $Var(X)$?

4. Suppose that X and Y are negatively correlated. Is $Var(X + Y)$ larger or smaller than $Var(X - Y)$?