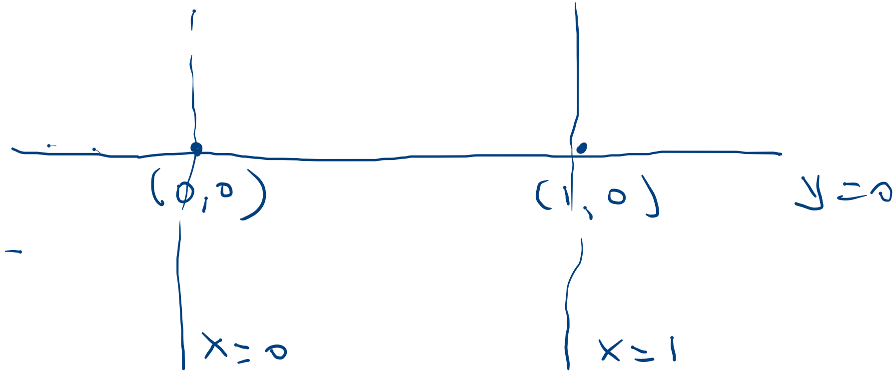
$\mathbb{R}$: $\mathbb{V}(P)$ → πρώτο

→ όχι ανάγωγο



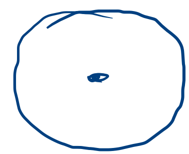
$f_1(x,y) = y$

$f_2(x,y) = x(x-1)$

$$\mathbb{V}(\langle y, x(x-1)\rangle) = \{(0,0),(1,0)\}$$

$$= \mathbb{V}(\langle y^2 + x^2(x-1)^2\rangle)$$

$\mathbb{V}(x^2+y^2-1)$ ανάγωγο $\iff$ $\mathbb{I}\,\mathbb{V}(x^2+y^2-1)$ πρώτο



Φυλλάδιο 1 β : $V = \mathbb{V}(\langle x^2-y^2,\ x^3+xy^2-y^3-x^2y-x+y\rangle)$

$$x^2-y^2 = (x-y)(x+y) \quad \begin{matrix} 1 \\ 2 \end{matrix}$$

$$x^3+xy^2-y^3-x^2y-x+y = (x^3-y^3)+(xy^2-x^2y)-(x-y) =$$

$$= (x-y)(x^2+xy+y^2-xy-1) \quad \begin{matrix} 1' \\ 2' \end{matrix}$$

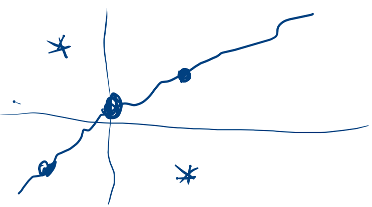
1, 1' : $x-y=0$ $\rightsquigarrow \mathbb{V}(x-y)$ (α)

1, 2' : $x-y=0,\ x^2+xy+y^2-xy-1=0$: $x-y=0,\ 2x^2-1=0 \Rightarrow x=\pm 1/\sqrt{2}$ (β)

2, 1' : $x-y=0,\ x+y=0$: $x=y=0$ (γ)

2, 2' : $x+y=0,\ x^2+xy+y^2-xy-1=0$: $x+y,\ 2x^2-1 \Rightarrow x=\pm 1/\sqrt{2}$ (δ)

$$V = \mathbb{V}(x-y) \cup \{\cancel{(1/\sqrt{2}, 1/\sqrt{2})}, \cancel{(-1/\sqrt{2}, -1/\sqrt{2})}\} \cup \{\cancel{(0,0)}\} \cup \{(1/\sqrt{2}, -1/\sqrt{2}), (-1/\sqrt{2}, 1/\sqrt{2})\}$$



$$V = \mathbb{V}(x-y) \cup \{(1/\sqrt{2}, -1/\sqrt{2})\} \cup \{(-1/\sqrt{2}, 1/\sqrt{2})\}.$$

$x-y$ είναι ανάγωγο πολ. άρα $\langle x-y\rangle$ πρώτο και $\mathbb{I}(\underbrace{\mathbb{V}(\langle x-y\rangle)}) = \langle x-y\rangle$ NSS, πρώτο

Φυλλάδιο 1, πρβλ. 2 : $V = \mathbb{V}(x^2-y^3,\ y^2-z^3) \subseteq \mathbb{R}^3$

α) $\varphi : \mathbb{R} \longrightarrow \mathbb{R}^3$ πολυωνυμική απεικ.

$t \longrightarrow (t^9, t^6, t^4)$.

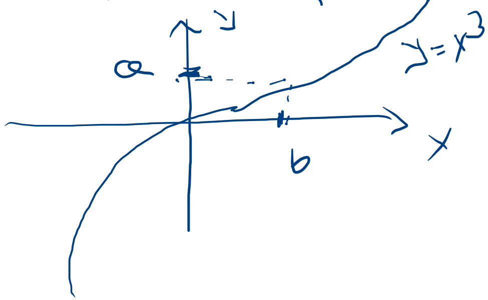
επάγει

$\varphi : \mathbb{R} \longrightarrow V$: $\varphi(\mathbb{R}) \subseteq V$ δηλ. τα

σημεία $(\underset{x}{t^9}, \underset{y}{t^6}, \underset{z}{t^4})$ ικανοποιούν το σύστημα $\begin{array}{l} x^2-y^3=0 \\ y^2-z^3=0 \end{array}$ V.

Είναι 1-1, επί : 1-1 $\varphi(t)=\varphi(t') \Longrightarrow t=t'$: $t^9 = t'^9 \Longrightarrow t=t'$

Επί: Στο $\mathbb{R}$: αν $a \in \mathbb{R}$ $\exists!$ b με $b^3=a$. : $y=x^3$



$$\Phi : \mathbb{R} \longrightarrow \mathbb{R}^3 \xrightarrow{f} \mathbb{R}$$

$$t \longrightarrow \underbrace{(t^9,\ t^6,\ t^4)}$$

$$\varphi : \mathbb{R} \longrightarrow V$$

$$\tilde{\varphi} : \mathbb{R}[V] \longrightarrow \mathbb{R}[t] \,.$$

Είναι ισομορφισμός ??

$$\mathbb{R}[V] = \frac{\mathbb{R}[x,y,z]}{I(V)}$$

$$f(x,y,z) \longmapsto f(t^9, t^6, t^4) \overset{?}{=} t \,.$$

$$f(x,y,z) = \sum a_{ijk}\, x^i y^j z^k$$

$$f(t^9, t^6, t^4) = \sum a_{ijk}\, \underbrace{t^{9i}\, t^{6j}\, t^{4k}}_{t^{9i+6j+4k} \neq t}, \quad i,j,k \geq 0$$

$$f(t)$$

$$I(V) = I\,V(\langle x^2 - y^3,\ y^2 - z^3 \rangle) = \langle x^2 - y^3,\ y^2 - z^3 \rangle \,.$$

$$= \ker \psi, \quad \psi : \mathbb{R}[x,y,z] \longrightarrow \mathbb{R}[t]$$ με $$f(x,y,z) \longmapsto f(t^9, t^6, t^4).$$

$\langle x^2-y^3, \underbrace{y^2-z^3} \rangle \subseteq \ker\varphi$.

Av $f \in k[x,y,z]$: $f(x,y,z) = q_1(x,y,z)(x^2-y^3) + r_1(y,z)x + r_0(y,z)$.

$k[y,z]$: $r_1(y,z) = q_2(y,z)(y^2-z^3) + r_1'(z)y + r_0'(z)$

$r_0(y,z) = q_3(y,z)(y^2-z^3) + r_1''(z)y + r_0''(z)$.

φ: $f = q_1(x^2-y^3) + (q_2 x + q_3)(y^2-z^3) + r_1'(z)xy + r_0'(z)x + r_1''(z)y + r_0''(z)$

Av $\underline{f \in \ker\varphi}$: Tobr $0 = f(t^9,t^6,t^4) = \underbrace{r_1'(t^4)}\, t^{15} + \underbrace{r_0'(t^4)}\, t^9 + \underbrace{r_1''(t^4)}\, t^6 + \underbrace{r_0''(t^4)}$

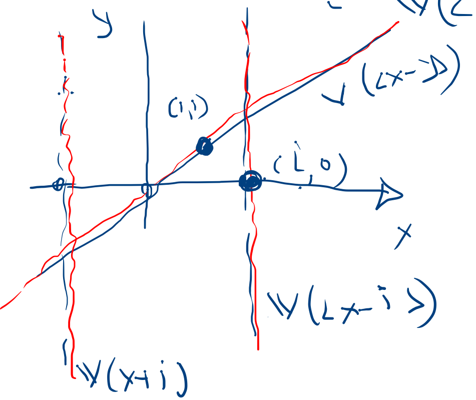
$f(t^9,t^6,t^4)=0,\ \forall t$ $\qquad \forall t$.

$$\left.\begin{aligned} 15 &\equiv 3 \mod 4 \\ 9 &\equiv 1 \mod 4 \\ 6 &\equiv 2 \mod 4 \\ 0 &\equiv 0 \mod 4. \end{aligned}\right\} \Rightarrow r_1' = r_0' = r_1'' = r_0'' = 0.$$

Φυλλ. 1 α) $V = \mathbb{V}(\langle x^3 + x - x^2y - y\rangle) = \mathbb{V}(\langle x^2(x-y) + (x-y)\rangle) =$

$= \mathbb{V}(\langle (x-y)(x^2+1)\rangle) = \underbrace{\mathbb{V}(x-y)}_{\text{αναγ.}} \cup \underbrace{\mathbb{V}(x^2+1)}_{?}$

$= \mathbb{V}(\langle x-y\rangle) \cup \mathbb{V}(\langle x-i\rangle) \cup \mathbb{V}(\langle x+i\rangle)$



Άσκηση 4β: $k[V]_{\bar{m}}$, $m = \langle x-1, y-1\rangle$.

(β)

$I(m) = \{(1,1)\}$

$$k[V]_{\bar{m}} = k[\mathbb{V}(\langle x-y\rangle)]_{\bar{m}} = \left(\mathbb{R}[x,y]/\langle x-y\rangle\right)_{\langle \overline{x-1}, \overline{y-1}\rangle} \cong (*)$$

$$\mathbb{R}[x,y]/\langle x-y\rangle \cong \mathbb{R}[t] \quad \left(\begin{matrix}\mathbb{R}[x,y] \to \mathbb{R}[t] \\ f(x,y) \to f(t,t)\end{matrix}\right)$$

(*) $\mathbb{R}[t]_{\langle t-1\rangle}$.

$\bar{m} = \langle \overline{x-i}, \bar{y}\rangle$